Optimal Investment and Premium Policies under Risk Shifting and Solvency Regulation

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Abstract

Risk shifting is a well-known agency problem in corporate finance which also exists between policyholders and shareholders of insurance companies. Shareholders engage in excessive risk taking at the expense of policyholders who, in turn, are less willing to pay for insurance coverage. Solvency regulation addresses this incentive problem by restricting the set of investment strategies and premium policies.

We first characterize Pareto optimal investment and premium policies and provide necessary and sufficient conditions for their existence and uniqueness. We then show that if shareholders cannot credibly commit to an investment strategy before policies are sold, they pursue an investment strategy that is either most risky or not risky at all. Last, we specify the conditions under which solvency regulation, such as Solvency II or the Swiss Solvency Test, mitigates the inefficiency of the risk shifting problem.

Keywords: Risk Shifting, Insurance, Regulation, Pareto Optimality

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1 Introduction

Risk shifting is a well-known agency problem in corporate finance between shareholders and creditors of a corporation (Jensen and Meckling [8]). The limited liability protection provides incentives for shareholders to overinvest in risky projects at the expense of creditors. If shareholders cannot credibly commit not to undertake those projects, creditors demand an appropriate interest rate differential which reflects the agency cost.

Policyholders of a stock insurance company face a situation similar to creditors of a corporation. By paying premiums, they provide capital which is senior to equity but under the investment discretion of shareholders who face limited liability. Policyholders are averse to excessive investment risk for two reasons. First, equivalent to creditors, they hold a short position of the default put option. Second, an increased risk of insolvency reduces the value policyholders discuss an agency problem in the context of dividend policies. After policies have been sold, shareholders have an incentive to increase the value of their claim by raising dividends at the expense of policyholders. They argue that the mutual organizational form of an insurance company can help internalize this agency problem. Doherty [6] shows how an increase in the risk structure of the insurer's asset portfolio increases the shareholders' position at the expense of policyholders. We contribute to this literature by providing a formal model to investigate this conflict of interest under the Pareto optimality condition, without and with the incentive problem, and examine the role of regulation in reducing the agency cost associated with this incentive problem.

We also contribute to the literature that analyzes the role of regulation in insurance markets. McCabe and Witt [12] develop a model for a monopolistic insurer that is exposed to insurance risk and market risk. The firm has to trade off risk and return under the regulatory constraint that the insolvency probability is below a certain threshold. While the firm is free to choose an insurance premium per standard exposure unit, the demand function for insurance is assumed to be independent of the firm's probability of insolvency. Borch [1] discusses the necessity of insurance regulation and argues it depends on the "patience" of shareholders. Regulation might only be necessary if shareholders are impatient and interested in quickly paying out dividends. Under CAPM assumptions, Munch and Smallwood [14] analyze the optimal amount of shareholder capital and investment risk of an insurance company. For the case with limited liability in an unregulated market, they impose the assumption that shareholders are restricted from investing in risky assets and that insurance losses are uncorrelated with the return of the market portfolio. Under these assumptions, the optimal amount of shareholder capital is either zero or infinitely large. They argue that, for zero shareholder capital and low insurance premiums, solvency regulation may reduce the probability of insolvency. Finsinger and Pauly [7] extend the model of Munch and Smallwood [14] by investigating the long run equilibrium of premiums and reserves. Still, shareholders are restricted from investing in risky assets and the premium paid by policyholders is independent of insolvency risk. They conclude that if market risk is uncorrelated with insurance risk, regulation of insurance companies may be unnecessary. MacMinn and Witt [9] develop a model for a risk-averse insurance company which sets the optimal investment policy in risky assets and underwriting activity that maximizes its expected utility of profits. Investment and insurance risk is assumed to be independent and premiums paid by policyholders are independent of insolvency risk. In this model, the authors discuss the impact of three regulatory schemes on optimal investment and underwriting activity. Although all of the above papers examine the impact of regulatory constraints on insurance markets, none develops a justification for regulation. Rees et al. [15], in fact, argue that if policyholders are fully informed about the insurer's insolvency risk then regulation serves no purpose.

We contribute to the papers above in several dimensions. In our model, shareholders have access to the stock market, policyholders are perfectly informed about the investment strategies of shareholders, and premiums therefore depend on the investment strategy and vice versa. Moreover, investment risk and insurance losses are allowed to be stochastically dependent. Then, without risk shifting and regulation, we characterize optimal investment strategies and premium policies based on the Pareto criterion. Furthermore, we introduce the risk shifting problem between shareholder and policyholders and characterize the solution without regulation. Last, this incentive problem justifies the regulatory role in insurance markets and we characterize the corresponding solution.

The paper is structured as follows. We set up our model in Section 2 and characterize the set of Pareto optimal policies in Section 3. In Section 4, we examine the risk shifting problem and solvency regulation. In Section 5, we calibrate our model to data and illustrate our results. We conclude in Section 6.

2 Setup

We consider a one-period economy with two agents, a policyholder and a shareholder. The policyholder is endowed with some initial wealth w_0 and faces a random loss X. His preferences are characterized by some Bernoulli utility function $u : \mathbb{R} \to \mathbb{R}$.

The shareholder is risk-neutral. He owns a stock insurance company with initial capital $c_0 > 0$ which offers full insurance coverage for X in exchange for a premium $p \geq 0$. The shareholder can invest a fraction $\alpha \in [0,1]$ of the total capital, $c_0 + p$, in the stock market which yields a random return R. The risk-free interest rate is assumed to be zero or, equivalently, all values are in units of the risk-free numeraire. The end of the period surplus is thus given by $(c_0 + p)(1 + \alpha R) - X$.

If the surplus is negative, the insurance company is insolvent and the shareholder is protected by limited liability. In this case, the policyholder receives the remaining assets, $(c_0+p)(1+\alpha R)$. Consequently, the terminal payoff to the shareholder equals

$$((c_0 + p)(1 + \alpha R) - X)^+$$

while the terminal wealth of the policyholder is given by

$$w_0 - p - (X - (c_0 + p)(1 + \alpha R))^+.$$

The corresponding utility of the shareholder and the policyholder as a function of α and p is

$$U_{SH}(\alpha, p) = \mathbb{E}\left[((c_0 + p)(1 + \alpha R) - X)^+\right]$$

and

$$U_{PH}(\alpha, p) = \mathbb{E}\left[u\left(w_0 - p - (X - (c_0 + p)(1 + \alpha R))^+\right)\right],$$

respectively.

Assumption 2.1. *Throughout the paper, we make the following standing assumptions:*

(i) *u is increasing, concave, and twice differentiable on $\mathbb{R}$ with $u' > 0$, $u'' < 0$, $\lim_{x\to-\infty} u(x) = -\infty$ and $\lim_{x\to-\infty} u'(x) = \infty$.*

(ii) *(X, R) admits a jointly continuous density function $f(x, r)$ in $\mathbb{R}^2$ with $\mathbb{E}[1 + \alpha R \mid X \le c_0 + p] > 0$ for all $(\alpha, p) \in [0, 1] \times \mathbb{R}_+$.*[1]

(iii) *The solvency event $S(\alpha, p) = \{(c_0 + p)(1 + \alpha R) \ge X\}$ has positive probability, $\mathbb{P}[S(\alpha, p)] > 0$, for all $(\alpha, p) \in [0, 1] \times \mathbb{R}_+$.*

(iv) *u and f are such that U_{SH} and U_{PH} are real-valued and differentiable in some neighborhood of $[0, 1] \times \mathbb{R}_+$, and the following formal manipulations (e.g. changing the order of differentiation and integration) are meaningful.*[2]

3 Optimal Investment and Premium Policies

In this setup, we now examine optimal investment and premium policies (α, p) in the following sense:

Definition 3.1. *The policy $(\alpha^*, p^*) \in [0, 1] \times \mathbb{R}_+$ is Pareto optimal if there does not exist any other policy $(\alpha, p) \in [0, 1] \times \mathbb{R}_+$ such that $U_{SH}(\alpha, p) \ge U_{SH}(\alpha^*, p^*)$ and $U_{PH}(\alpha, p) \ge U_{PH}(\alpha^*, p^*)$ with strict inequality for at least one of them.*

We first show that Pareto optimality is equivalent to a constrained optimization problem.

Theorem 3.2. *For any policy $(\alpha^*, p^*) \in [0, 1] \times \mathbb{R}_+$, the following are equivalent:*

(i) *(α^*, p^*) is Pareto optimal.*

(ii) *(α^*, p^*) solves the constrained optimization problem*

$$\begin{aligned} \max_{(\alpha,p)\in[0,1]\times\mathbb{R}_+} & \ U_{SH}(\alpha, p) \\ s.t. & \ U_{PH}(\alpha, p) \ge \gamma_{PH} \end{aligned} \tag{1}$$

for the policyholder's reservation utility level $\gamma_{PH} = U_{PH}(\alpha^*, p^*)$.

(iii) *(α^*, p^*) solves the constrained optimization problem*

$$\begin{aligned} \max_{(\alpha,p)\in[0,1]\times\mathbb{R}_+} & \ U_{PH}(\alpha, p) \\ s.t. & \ U_{SH}(\alpha, p) \ge \gamma_{SH} \end{aligned} \tag{2}$$

for the shareholder's reservation utility level $\gamma_{SH} = U_{SH}(\alpha^*, p^*)$.

Moreover, in either of the above optimization problems, (1) and (2), the respective reservation utility constraint is binding.

[1] This condition is needed in Lemma A.2. It is automatically satisfied if $R > -1$ a.s.

[2] For example, it is sufficient (but not necessary) that f has compact support.

Proof. (i) $\Rightarrow$ (ii): We argue by contradiction, and assume that there exists a policy $(\bar{\alpha}, \bar{p})$ such that $U_{SH}(\bar{\alpha}, \bar{p}) > U_{SH}(\alpha^*, p^*)$ and $U_{PH}(\bar{\alpha}, \bar{p}) \geq \gamma_{PH}$. Then (α^*, p^*) is not Pareto optimal.

(ii) $\Rightarrow$ (i): Again by contradiction, we assume that (α^*, p^*) is not Pareto optimal. Hence there exists some policy $(\bar{\alpha}, \bar{p})$ such that $U_{SH}(\bar{\alpha}, \bar{p}) \geq U_{SH}(\alpha^*, p^*)$ and $U_{PH}(\bar{\alpha}, \bar{p}) \geq \gamma_{PH}$ with strict inequality for at least one of them. If $U_{SH}(\bar{\alpha}, \bar{p}) > U_{SH}(\alpha^*, p^*)$ then clearly (α^*, p^*) does not solve (1). If $U_{PH}(\bar{\alpha}, \bar{p}) > \gamma_{PH}$ then there exists a neighborhood O of $(\bar{\alpha}, \bar{p})$ such that $U_{PH}(\alpha, p) \geq \gamma_{PH}$ for all $(\alpha, p) \in O$. Moreover, by Lemma A.2 below, $\nabla U_{SH}(\bar{\alpha}, \bar{p}) \neq 0$. We conclude that there exists some $(\alpha, p) \in O$ with $U_{SH}(\alpha, p) > U_{SH}(\bar{\alpha}, \bar{p}) \geq U_{SH}(\alpha^*, p^*)$. Hence, again, (α^*, p^*) does not solve (1). Moreover, this shows that the reservation utility constraint is binding.

The equivalence (i) $\Leftrightarrow$ (iii) follows analogously. $\square$

As for the existence and uniqueness of Pareto optimal policies, we have the following two theorems.

Theorem 3.3. *For any reservation utility level, $\gamma_{PH} \in U_{PH}([0,1] \times \mathbb{R}_+)$ or $\gamma_{SH} \in U_{SH}([0,1] \times \mathbb{R}_+)$, respectively, there exists at least one Pareto optimum $(\alpha^*, p^*) \in [0,1] \times \mathbb{R}_+$. It satisfies the first order condition*

$$\mathbb{E}\left[R\, u'\left(w_0 - p^* - (X - (c_0+p^*)(1+\alpha^* R))^+\right)\right] \begin{cases} \leq 0, & \text{if } \alpha^* = 0, \\ = 0, & \text{if } 0 < \alpha^* < 1, \\ \geq 0, & \text{if } \alpha^* = 1. \end{cases} \tag{3}$$

Moreover, for any $\alpha^* \in [0,1]$ there exists at most one Pareto optimum.

Proof. In view of Theorem 3.2, it is enough to consider the optimization problem (1). Denote by $C = \{U_{PH} \geq \gamma_{PH}\} \subset [0,1] \times \mathbb{R}_+$ the constraint set. For existence of a Pareto optimum, we show that C is compact. Suppose, by contradiction, there exists a sequence (α_n, p_n) in C with $p_n \geq n$ for all $n \in \mathbb{N}$. Then

$$U_{PH}(\alpha_n, p_n) \leq \mathbb{E}\left[u\left(w_0 - n\right)\right] \to -\infty \quad \text{for } n \to \infty, \tag{4}$$

which contradicts the fact that $\gamma_{PH} \in \mathbb{R}$. Thus, since U_{SH} is continuous on $[0,1] \times \mathbb{R}_+$, it attains a maximum on C.

Moreover, in view of Lemma A.2, $U_{SH}(\alpha, p)$ is strictly increasing in p, for all $\alpha \in [0,1]$. Hence, for any fixed $\alpha \in [0,1]$, there can be at most one Pareto optimum.

For the derivation of the first order condition, it is convenient to introduce the following diffeomorphism:

$$\begin{aligned} [0,1] \times \mathbb{R}_+ &\to \{(v,w) \mid 0 \leq v \leq w,\ w \in [c_0, \infty)\} \\ (\alpha, p) &\mapsto (v,w) = ((c_0+p)\alpha, c_0+p). \end{aligned}$$

Note that w is the total asset value of the insurer and v is the money invested in the stock market. The corresponding utility of the shareholder and the policyholder as a function of the new coordinates (v, w) is

$$\begin{aligned} V_{SH}(v,w) &= \mathbb{E}\left[(w + vR - X)^+\right] \\ V_{PH}(v,w) &= \mathbb{E}\left[u\left(w_0 + c_0 - w - (X - w - vR)^+\right)\right], \end{aligned} \tag{5}$$

so that $V_{SH}(v,w) = U_{SH}(\alpha,p)$ and $V_{PH}(v,w) = U_{PH}(\alpha,p)$. For simplicity of notation, we use the same letter $\mathsf{S}(v,w) = \mathsf{S}(\alpha,p)$ for the respective solvency event.

We note, en passant, that V_{SH} is a convex and V_{PH} is a concave function jointly in (v,w). In contrast, U_{SH} and U_{PH} do not share these properties as functions jointly in (α,p) in general (see Figure 4 in Appendix B).

As seen in (4) above, we have $\lim_{w\to\infty} V_{PH}(v,w) = -\infty$ for any $v \in \mathbb{R}_+$, and, by Lemma A.3, $\partial_w V_{PH} < 0$. Hence, for any fixed $\gamma_{PH} \in V_{PH}(\{(v,w) \mid 0 \leq v \leq w,\, w \in [c_0, \infty)\})$, the implicit function theorem yields a continuously differentiable function $W : I \to [c_0, \infty)$ on some interval $I \subseteq \mathbb{R}_+$ with $v \leq W(v)$, $V_{PH}(v, W(v)) = \gamma_{PH}$, and

$$W'(v) = -\frac{\partial_v V_{PH}(v,W(v))}{\partial_w V_{PH}(v,W(v))} = \frac{\mathbb{E}\left[R\, u'(w_0 + c_0 - X + vR)\, 1_{\mathsf{S}(v,W(v))^c}\right]}{u'(w_0 + c_0 - W(v))\mathbb{P}\left[\mathsf{S}(v,W(v))\right]}.$$

We now characterize the critical points for the shareholder utility function along the level curve $(v, W(v))$. A calculation shows

$$\begin{aligned} \frac{d}{dv} V_{SH}(v,W(v)) &= \partial_v V_{SH}(v,W(v)) + W'(v)\partial_w V_{SH}(v,W(v)) \\ &= -\mathbb{E}\left[R\, 1_{\mathsf{S}(v,W(v))}\right] + \frac{\mathbb{E}\left[R\, u'(w_0 + c_0 - X + vR)\, 1_{\mathsf{S}(v,W(v))^c}\right]}{u'(w_0 + c_0 - W(v))} \\ &= -\frac{\mathbb{E}\left[R\, u'\left(w_0 + c_0 - W(v) - (X - W(v) - vR)^+\right)\right]}{u'(w_0 + c_0 - W(v))}. \end{aligned} \tag{6}$$

Hence any Pareto optimal $(v^*, w^*) \in \{(v,w) \mid 0 \leq v \leq w,\, w \in [c_0, \infty)\}$ satisfies

$$\mathbb{E}\left[R\, u'\left(w_0 + c_0 - w^* - (X - w^* - v^* R)^+\right)\right] \begin{cases} \geq 0, & \text{if } v^* = w^*, \\ = 0 & \text{if } 0 < v^* < w^*, \\ \leq 0, & \text{if } v^* = 0. \end{cases}$$

This proves (3). $\square$

Theorem 3.4. *Assume that*

$$\mathbb{E}\left[R \mid X \geq c_0 + p\right] > 0 \quad \textit{for all } p \in \mathbb{R}_+. \tag{7}$$

Then, for any reservation utility level, $\gamma_{PH} \in U_{PH}([0,1] \times \mathbb{R}_+)$ *or* $\gamma_{SH} \in U_{SH}([0,1] \times \mathbb{R}_+)$*, respectively, there exists a unique Pareto optimum* $(\alpha^*, p^*) \in [0,1] \times \mathbb{R}_+$*. Moreover, the first order condition* (3) *is also sufficient for Pareto optimality of* $(\alpha^*, p^*) \in [0,1] \times \mathbb{R}_+$.

Remark 3.5. *Assumption* (7) *states that the conditional expected return of the stock market given small insurance losses is positive. From a risk management point of view, it is important to note that this moderate assumption is compatible with stress scenarios of negative expected returns under catastrophic insurance losses. A sufficient condition for* (7) *is*

$$\mathbb{E}\left[R \mid X = x\right] > 0 \quad \textit{for all } \operatorname{ess\,inf} X < x < \operatorname{ess\,sup} X.$$

In particular, this sufficient condition holds if the stock market has a positive unconditional expected return, $\mathbb{E}[R] > 0$*, and* R *and* X *are independent.*

Proof. First, observe that (7) is equivalent to

$$\mathbb{E}\left[R\, 1_{\mathcal{S}(\alpha;p)}\right] > 0 \quad \text{for all } (\alpha, p) \in [0,1] \times \mathbb{R}_+. \tag{8}$$

We continue our proof using the (v, w)-coordinates as introduced in the proof of Theorem 3.3. Fix $\gamma_{PH} \in V_{PH}(\{(v,w) \mid 0 \leq v \leq w, w \in [c_0, \infty)\})$, and let $W : I \to [c_0, \infty)$ be the corresponding level curve as in the proof of Theorem 3.3. Suppose $v^* \in I$ is a critical point, $\frac{d}{dv} V_{SH}(v^*, W(v^*)) = 0$. Then, in view of (6), the second derivative of $V_{SH}(v, W(v))$ at v^* equals[3]

$$\begin{aligned}\frac{d^2}{dv^2} V_{SH}(v, W(v))|_{v=v^*} = & -W'(v^*) \frac{u''(w_0 + c_0 - W(v^*))\, \mathbb{E}\left[R\, 1_{\mathcal{S}(v^*;W(v^*))}\right]}{u'(w_0 + c_0 - W(v^*))} \\ & + \frac{\mathbb{E}\left[R^2\, u''(w_0 + c_0 - X + v^* R)\; 1_{\mathcal{S}(v^*;W(v^*))^c}\right]}{u'(w_0 + c_0 - W(v^*))}.\end{aligned}$$

In view of Lemma A.3 and (8), we know that $\partial_v V_{SH} > 0$ and $\partial_w V_{SH} > 0$. By the first equality in (6), we infer that $W'(v^*) < 0$, and thus

$$\frac{d^2}{dv^2} V_{SH}(v, W(v))|_{v=v^*} < 0.$$

Hence any critical point $v^* \in I$ is a local maximum, and thus a global maximum in I. We conclude that either $V_{SH}(v, W(v))$ is strictly increasing in I, or strictly decreasing in I, or attains a global maximum at a unique critical point v^* in I and is strictly increasing to the left and strictly decreasing to the right of v^*. This proves the theorem. □

4 Risk Shifting and Solvency Regulation

We now examine the situation in which the shareholder cannot credibly commit to an investment policy α before the insurance premium p is paid. This incentive problem, also known as risk shifting problem, is typically discussed in the context of the financing of a corporation by shareholders and debt holders (see Jensen and Meckling [8]).

[3] The differentiation under the expectation sign is justified by Assumption 2.1, see the proof of Lemma A.2 below.

In our setup, this can be formalized by the following constrained optimization problem:

$$
\begin{aligned}
\max_{(\alpha,p)\in[0,1]\times\mathbb{R}_+} \quad & U_{SH}(\alpha,p) \\
\text{s.t.} \quad & U_{PH}(\alpha,p) \geq \gamma_{PH}, \\
& \alpha \in \arg\max_{\alpha'} U_{SH}(\alpha',p)
\end{aligned}
\tag{9}
$$

for some policyholder's reservation utility level γ_{PH}.

In view of (1), any solution of the risk shifting problem (9) is generically Pareto suboptimal. Moreover, it is characterized as follows.

Theorem 4.1. *For any policyholder's reservation utility level γ_{PH} there exists at most two solutions $(\bar\alpha,\bar p)$ in $[0,1]\times\mathbb{R}_+$ to (9). Any such solution satisfies $U_{PH}(\bar\alpha,\bar p)=\gamma_{PH}$ and $\bar\alpha\in\{0,1\}$ along with the first order condition*

$$
\mathbb{E}\left[R\,1_{\mathcal{S}(\bar\alpha,\bar p)}\right]
\begin{cases}
\leq 0, & \text{if } \bar\alpha = 0,\\
\geq 0, & \text{if } \bar\alpha = 1.
\end{cases}
\tag{10}
$$

Moreover, under the policyholder's outside option of not buying insurance, that is, $\gamma_{PH}=\mathbb{E}[u(w_0-X)]$, there exists a solution.

Proof. By Lemma A.1, for any fixed $p\in\mathbb{R}_+$, we have $\arg\max_{\alpha'} U_{SH}(\alpha',p)\subseteq\{0,1\}$. Moreover, the corresponding first order condition is

$$
\partial_\alpha U_{SH}(\alpha,p)
\begin{cases}
\leq 0, & \text{if } \alpha = 0 \in \arg\max_{\alpha'} U_{SH}(\alpha',p),\\
\geq 0, & \text{if } \alpha = 1 \in \arg\max_{\alpha'} U_{SH}(\alpha',p).
\end{cases}
$$

Hence $\bar\alpha\in\{0,1\}$, and (10) follows from Lemma A.2. Further, by Lemma A.2 again, $\partial_p U_{SH}(\alpha,p)>0$ for all $\alpha\in[0,1]$. Hence, for any fixed $\alpha\in[0,1]$, there exists at most one solution of $\max_{p\in\mathbb{R}_+} U_{SH}(\alpha,p)$. We conclude that there exists at most two solutions to (9), and the policyholder's reservation utility constraint is binding. The last statement follows since $U_{PH}(\alpha,0)\geq\mathbb{E}[u(w_0-X)]$ and $\lim_{p\to\infty} U_{PH}(\alpha,p)=-\infty$, for all $\alpha\in[0,1]$. □

It is interesting to note that $\bar\alpha=0$ is a possible solution to the risk shifting problem in general. This has to do with the fact that the shareholder's limited liability put option has a random strike, X, which can be correlated with R. Intuitively, if the stock market risk R provides a hedge against insurance losses X, then investing in the stock market might actually reduce the overall risk. However, under the more stringent, but realistic assumption (7), see Remark 3.5, we infer:

Corollary 4.2. *If (7) holds, then the only possible solution $(\bar\alpha,\bar p)$ to the risk shifting problem (9) is attained for $\bar\alpha=1$. Moreover, $(1,\bar p)$ is Pareto optimal if and only if*

$$
\mathbb{E}\left[R\,u'\left(w_0-\bar p-(X-(c_0+\bar p)(1+R))^+\right)\right] \quad 0.
$$

Proof. This follows from (8), (10), and Theorem 3.4. □

We now examine how solvency regulation can help mitigate the inefficiency of the investment and premium policy implied by the risk shifting problem. The regulator assesses the riskiness of the annual loss

$$L(\alpha, p) = c_0 - ((c_0 + p)(1 + \alpha R) - X) = -(c_0 + p)\alpha R + X - p$$

by means of a risk measure ρ. The respective regulatory requirement is that the available capital, c_0, be greater than the required capital, $\rho\,(L(\alpha, p))$:

$$\rho\,(L(\alpha, p)) \leq c_0.$$

The risk shifting problem under this additional regulatory constraint is as follows

$$\begin{aligned} \max_{(\alpha, p)\in[0,1]\times\mathbb{R}_+} \; & U_{SH}(\alpha, p) \\ \text{s.t. } & U_{PH}(\alpha, p) \geq \gamma_{PH}, \\ & \alpha \in \arg\max_{\alpha'} U_{SH}(\alpha', p) \\ & \qquad \text{s.t. } \rho(L(\alpha', p)) \leq c_0 \end{aligned} \tag{11}$$

for some policyholder's reservation utility level γ_{PH}.

Assumption 4.3. *Throughout, ρ satisfies the following conditions:*

(i) *ρ is cash-invariant, that is, $\rho(L + c) = \rho(L) + c$ for any constant cash amount $c \in \mathbb{R}$ and random loss L.*

(ii) *ρ is convex, that is,*

$$\rho(\lambda L + (1 - \lambda)L') \leq \lambda\rho(L) + (1 - \lambda)\rho(L')$$

for all $\lambda \in [0, 1]$ and random losses L, L'.

(iii) *$\rho(L(\alpha, p))$ is continuous in $\alpha \in [0, 1]$ for all $p \in \mathbb{R}_+$.*

These are standard assumptions for risk measures, see e.g. McNeil et al. [13]. The cash-invariance property of ρ is motivated by its interpretation as regulatory capital requirement. Adding a deterministic cash amount c to the position, the capital requirement is reduced by the same amount.

The convexity of ρ is crucial for the constrained problem (11) to be well-posed. See also Remark 4.5 below.

Property (iii) is of technical nature. It implies that

$$\alpha\,(p) = \sup\{\alpha \in [0, 1] \mid \rho(L(\alpha, p)) \leq c_0\}$$

satisfies either $\rho(L(\alpha\,(p), p)) = c_0$, or $\alpha\,(p) = 1$ if $\rho(L(\alpha, p)) \leq c_0$ for all $\alpha \in [0, 1]$, or $\alpha\,(p) = -\infty$ if $\rho(L(\alpha, p)) > c_0$ for all $\alpha \in [0, 1]$.[4] If the shareholder

[4] Following the usual convention, we define $\sup \emptyset = -\infty$.

prefers great α, that is, $\partial_\alpha U_{SH} > 0$, then $\alpha^*(p)$ equals the arg max in the regulatory constrained subproblem in (11) given that it is well-posed, that is, $\alpha^*(p) > -\infty$.

Here is our existence and uniqueness result for the regulatory constrained risk shifting problem (11).

Theorem 4.4. *Suppose* (7) *holds. Then, for any policyholder's reservation utility level* $\gamma_{PH} \in \{U_{PH}(\alpha^*(p), p) \mid \alpha^*(p) > -\infty, p \in \mathbb{R}_+\}$*, there exists a unique solution* $(\hat{\alpha}, \hat{p})$ *in* $[0,1] \times \mathbb{R}_+$ *to* (11)*. It satisfies* $U_{PH}(\hat{\alpha}, \hat{p}) = \gamma_{PH}$*. Moreover,* $\rho(L(\hat{\alpha}, \hat{p})) = c_0$ *if* $\hat{\alpha} < 1$.

Proof. We argue in the (v, w)-coordinates introduced in the proof of Theorem 3.3, and define the corresponding regulator's risk measurement function

$$V_R(v, w) = \rho(-w - vR + X) = -w + \rho(-vR + X)$$

for $0 \le v \le w$, $w \in [c_0, \infty)$. It follows by inspection that $V_R(v, w) = \rho(L(\alpha, p)) - c_0$, and thus $V_R(v, w)$ is jointly continuous in (v, w). Moreover, $\alpha^*(p)$ corresponds to

$$v^*(w) = \sup\{v \in [0, w] \mid V_R(v, w) \le 0\}.$$

In view of Lemma A.3 and (8), we know that $\partial_v V_{SH} > 0$. Hence, any solution $(\hat{v}, \hat{w})$ to (11) must be of the form $\hat{v} = v^*(\hat{w})$.

We claim that v^* is a non-decreasing function on $[c_0, \infty)$. Indeed, arguing by contradiction, suppose that $v^*(w') < v^*(w)$ for some $w' > w$. Then $v^*(w) \in [0, w]$ and there exists some $v \in (v^*(w'), v^*(w)] \neq \emptyset$ with $V_R(v, w) \le 0$. By cash-invariance of ρ it follows $V_R(v, w') = V_R(v, w) + w - w' < V_R(v, w) \le 0$, which again implies $v \le v^*(w')$, a contradiction. Whence v^* is non-decreasing.

Moreover, since ρ and thus V_R is convex, the function $v^* : [c_0, \infty) \to [-\infty, \infty)$ is concave, and thus continuous on the interior of its domain (see e.g. Rockafellar [16, Theorems 5.3 and 10.1]).

By assumption, there exists some $\underline{w} \in [c_0, \infty)$ with $v^*(\underline{w}) \in [0, \underline{w}]$ and $V_{PH}(v^*(\underline{w}), \underline{w}) \ge \gamma_{PH}$. Since also $\partial_w V_{SH} > 0$, we conclude that $V_{SH}(v^*(w), w)$ is a strictly increasing continuous function in $w \in [\underline{w}, \infty)$. As shown in (4), the policyholder's constraint set $\{V_{PH} \ge \gamma_{PH}\}$ is compact in $\{(v, w) \mid 0 \le v \le w, w \in [c_0, \infty)\}$. Hence $V_{SH}(v^*(w), w)$ attains a unique maximum in some $\hat{w} \in [\underline{w}, \infty)$, and $V_{PH}(v^*(\hat{w}), \hat{w}) = \gamma_{PH}$. Whence the theorem is proved. □

Remark 4.5. *We note that without convexity of* ρ*, the function* $v^*(w)$ *and thus* $V_{SH}(v^*(w), w)$ *may fail to be continuous in* w*. Therefore the maximum of* (11) *may not be attained.*

Whether or not the regulatory constraint helps to mitigate the inefficiency of the risk shifting problem can now be decided from case to case. Since, in view of Theorems 4.1 and 4.4, under the assumption of (7), the policyholder's reservation utility constraint is binding in either case, it is enough to compare the shareholder's utility as a function of $\alpha \in [0, 1]$ along the policyholder's respective level curve. As we have seen at the end of the proof of Theorem 3.4, this function is either strictly increasing, or strictly decreasing, or attains a global maximum at a unique critical point α^* *2* $[0, 1]$ and is strictly increasing to the left and strictly decreasing to the right of α^*.

If the shareholder's utility along the policyholder's respective level curve is strictly increasing in α, then

$$\hat{\alpha} \quad \alpha^* = \bar{\alpha} = 1.$$

Hence, solvency regulation can only be harmful.

If the shareholder's utility along the policyholder's respective level curve is strictly decreasing in α, then

$$0 = \alpha^* \quad \hat{\alpha} \quad \bar{\alpha} = 1.$$

Thus, regulation mitigates the inefficiency of the risk shifting problem.

Last, if the shareholder's utility along the policyholder's respective level curve attains a global maximum at a unique critical point α^* *2* $(0, 1)$ and is strictly increasing to the left and strictly decreasing to the right of α^*, then either

$$0 < \alpha^* \quad \hat{\alpha} \quad \bar{\alpha} = 1$$

or

$$0 \quad \hat{\alpha} \quad \alpha^* < \bar{\alpha} = 1.$$

In the first case, since the shareholder's utility is strictly decreasing for all $\alpha \quad \alpha^*$, regulation mitigates the inefficiency of the risk shifting problem. In the second case, the impact of the regulatory constraint is ambiguous. In particular, a very tight solvency regulation might turn out to be harmful relative to the risk shifted solution.

5 Numerical Example

We calibrate our model to an European Economic Area non-life insurer average portfolio taken from the QIS3 (Quantitative Impact Study 3) Benchmarking Study [5] of the Chief Risk Officer (CRO) Forum. The stand alone capital requirements[5] for stock market investment and insurance risk under the Solvency II standard model [3, 4] are

$$k_{mkt} = 2508, \quad k_{ins} = 4332.$$

Under Solvency II, the market investment and insurance risk are assumed to have a linear correlation coefficient of 0.25. That is, the diversified total solvency capital requirement equals

$$k_{tot} = \sqrt{k_{mkt}^2 + 2 \quad 0.25 \quad k_{mkt}\, k_{ins} + k_{ins}^2} = 5522.$$

[5] These gures are derived from the proportion splits of QIS3 capital charges as shown on pages 39, 41, 43 in the document [5]. The capital requirements are thus normalized such that the undiversi ed total solvency capital requirement (SCR) results in 100×100. The risk class \default" is negligible and the market risk types other than \equity" have been omitted for simplicity.

The Solvency II risk measure ρ is the value-at-risk, $\mathrm{VaR}_{99:5\%}$, at the 99.5% confidence level. Thus, the Solvency II stand alone capital requirement for market risk is

$$k_{mkt} = \mathrm{VaR}_{99:5\%}\,[\text{market loss} = \quad (c_0 + p_0)\,\alpha_0\,R] \tag{12}$$

for some representative premium p_0 to be determined below, and the representative investment policy $\alpha_0 = 1/7$.[6] The Solvency II stand alone capital requirement for insurance risk equals

$$k_{ins} = \mathrm{VaR}_{99:5\%}\,[\text{insurance loss} = X \quad p_0]\,. \tag{13}$$

Moreover, we assume that available equals required capital

$$k_{tot} = c_0. \tag{14}$$

It remains to specify the stochastic model for market investment and insurance risk. We assume that

$$R = \mathrm{e}^{Y} \quad 1 \quad \text{and} \quad X = \mathrm{e}^{Z},$$

where (Y, Z) is jointly normal distributed with mean (μ_Y, μ_Z), standard deviations σ_Y, σ_Z, and linear correlation of $\quad 0.25$. This yields an approximate linear correlation of 0.25 between $\quad R$ and X.

We calibrate the market risk parameters according to

$$\mathbb{E}[R] = 0.04, \quad \mathrm{var}[R] = 0.16^2. \tag{15}$$

This determines μ_Y and σ_Y. The insurance risk parameters are assumed to satisfy the following premium calculation principle

$$p_0 = \mathbb{E}[X] + \sqrt{\mathrm{var}[X]}. \tag{16}$$

Equations (12)–(16) determine the model parameters μ_Y, μ_Z, σ_Y, σ_Z.

As for the policyholder's utility function, we assume constant absolute risk aversion, that is, $u(x) = \quad \mathrm{e}^{-\ \ x}$ and thus

$$U_{PH}(\alpha, p) = \quad \mathrm{e}^{-\ \ w_0}\,\mathbb{E}\left[\mathrm{e}^{-\ \ \left(-p-(X-(c_0+p)(1+\ \ R))^+\right)}\right].$$

By Lemma A.4, Assumption (7) is satisfied. In our numerical example, Theorem 3.4, Corollary 4.2, and Theorem 4.4 imply that there exists a unique Pareto optimal policy (α^*, p^*) in $[0,1] \quad \mathbb{R}_+$, the risk shifted solution $(\bar{\alpha}, \bar{p})$ is attained at $\bar{\alpha} = 1$, and there exists a unique solution $(\hat{\alpha}, \hat{p})$ in $[0,1] \quad \mathbb{R}_+$ for the regulated risk shifting problem.

We note that the value-at-risk, $\mathrm{VaR}_{99:5\%}$, does not satisfy the convexity property in Assumption 4.3 in general, see e.g. McNeil et al. [13]. However, in our example it shows convex behavior for the relevant values of (α, p). For

[6] This number is derived from annual nancial statements of non-life insurers.

comparison, we also consider the Swiss Solvency Test [2] regulatory risk measure, which is the expected shortfall[7], $\mathrm{ES}_{99\%}$, at the 99% confidence level. Expected shortfall satisfies all properties of Assumption 4.3.

For different degrees of risk aversion, $\beta \in \{10, 30, 70\}$, Figures 1–3 in Appendix B depict our numerical results in (α, p) where α is on the horizontal and p on the vertical axis. The indifference curves of the shareholder and policyholder for different reservation utility levels γ_{SH} and γ_{PH} are labeled "IC Sh" and "IC Ph", respectively. The thick line plots the policies (α, p) that satisfy the first order condition (3) for Pareto optimality, i.e. the tangency points of the shareholder's and policyholder's indifference curves. The dotted and slash-dotted lines are the boundaries of the regulatory constraints $\rho\left(L(\alpha, p)\right) \leq c_0$ under the value-at-risk measure $\mathrm{VaR}_{99.5\%}$ and the expected shortfall measure $\mathrm{ES}_{99\%}$, respectively. The policies that are acceptable to the regulator are to the north-west of these boundaries. The asterisk is the Pareto optimal policy (α^*, p^*) under the policyholder's outside option of not buying insurance, that is, $\gamma_{PH} = \mathbb{E}\left[u(w_0 - X)\right]$.

We observe that, with increasing degree of risk aversion β, the optimal investment α^* in the stock market reduces while the optimal premium level p^* increases. Moreover, in our example, the expected shortfall measure implies a more stringent regulatory requirement than the one implied by the value-at-risk measure. For lower degrees of risk aversion, $\beta = 10$ and $\beta = 30$, the Pareto optimal policies (α^*, p^*) do not satisfy the regulatory constraints. For higher degrees of risk aversion, e.g. $\beta = 70$, the regulatory requirements are fulfilled for the policy (α^*, p^*).

Figure 4 in Appendix B shows a larger section of the (α, p)-plane for $\beta = 30$. By A, B, C, and D we denote the Pareto optimal policy, the risk shifted solution, and the two regulated risk shifted solutions, respectively.

Table 1 shows the values of the policy (α, p) and the implied utility of the shareholders at the Pareto optimal policy, A, the risk shifted solution, B, and the risk shifted solutions under the two regulatory regimes, C and D. We observe that solvency regulation improves efficiency in our example under the risk shifting problem.

6 Conclusion

We investigate the following trade-off that shareholders of insurance companies face when determining their investment strategy. On the one hand, a riskier strategy increases the value of their position due to the limited liability protection. On the other hand, it reduces the capital available for investment, and thereby the value of their position, since it reduces the premium policyholders are willing to pay due to the increased insolvency probability.

We characterize investment and premium policies under Pareto optimality, under the risk shifting problem, and under solvency regulation. In particular, we specify the conditions under which solvency regulation, by limiting the set

[7] Also known as conditional or tail value-at-risk.

of possible investment and premium policies, reduces the inefficiency caused by the risk shifting problem. However, we point out that very restrictive regulation might actually backfire.

A Appendix: Lemmas

Lemma A.1. *The shareholder's utility function $U_{SH}(\alpha, p)$ is convex in p (in α) for a fixed α (fixed p). The policyholder's utility function $U_{PH}(\alpha, p)$ is concave in p (in α) for a fixed α (fixed p).*

Proof. For fixed $R = r$ and $X = x$, the function $((c_0 + p)(1 + \alpha R) - X)^+$ is convex, and $u(w_0 - p - (X - (c_0 + p)(1 + \alpha R))^+)$ is concave, in p (in α) for a fixed α (fixed p). Taking expectation preserves these properties. □

Lemma A.2. *The derivatives of U_{SH} and U_{PH} are given by:*

$$\begin{aligned}
\partial_\alpha U_{SH}(\alpha, p) &= (c_0 + p)\mathbb{E}\left[R\, 1_{\mathcal{S}(\alpha;p)}\right] \\
\partial_p U_{SH}(\alpha, p) &= \mathbb{E}\left[(1 + \alpha R)\, 1_{\mathcal{S}(\alpha;p)}\right] > 0 \\
\partial_\alpha U_{PH}(\alpha, p) &= (c_0 + p)\mathbb{E}\left[R\, u'(w_0 - p - X + (c_0 + p)(1 + \alpha R))\, 1_{\mathcal{S}(\alpha;p)^c}\right] \\
\partial_p U_{PH}(\alpha, p) &= -u'(w_0 - p)\mathbb{P}\left[\mathsf{S}(\alpha, p)\right] \\
&\quad + \alpha\mathbb{E}\left[R\, u'(w_0 - p - X + (c_0 + p)(1 + \alpha R))\, 1_{\mathcal{S}(\alpha;p)^c}\right]
\end{aligned}$$

for all $(\alpha, p) \in [0, 1] \times \mathbb{R}_+$.

Proof. We can write

$$\begin{aligned}
U_{SH}(\alpha, p) &= \int_{-\infty}^{\infty}\int_{-\infty}^{(c_0+p)(1+\alpha r)} ((c_0 + p)(1 + \alpha r) - x)\, f(x, r)\, dx\, dr \\
U_{PH}(\alpha, p) &= \int_{-\infty}^{\infty}\int_{-\infty}^{(c_0+p)(1+\alpha r)} u(w_0 - p) f(x, r)\, dx\, dr \\
&\quad + \int_{-\infty}^{\infty}\int_{(c_0+p)(1+\alpha r)}^{\infty} u(w_0 - p - x + (c_0 + p)(1 + \alpha r)) f(x, r)\, dx\, dr
\end{aligned}$$

Hence the assertion follows by straightforward formal differentiation as justified by Assumption 2.1. That $\partial_p U_{SH}(\alpha, p) > 0$ follows from Assumption 2.1(ii). □

Lemma A.3. *The derivatives of V_{SH} and V_{PH} defined in* (5) *are given by:*

$$\begin{aligned}
\partial_v V_{SH}(v, w) &= \mathbb{E}\left[R\, 1_{\mathcal{S}(v;w)}\right] \\
\partial_w V_{SH}(v, w) &= \mathbb{P}\left[\mathsf{S}(v, w)\right] > 0 \\
\partial_v V_{PH}(v, w) &= \mathbb{E}\left[R\, u'(w_0 + c_0 - X + vR)\, 1_{\mathcal{S}(v;w)^c}\right] \\
\partial_w V_{PH}(v, w) &= -u'(w_0 + c_0 - w)\mathbb{P}\left[\mathsf{S}(v, w)\right] < 0
\end{aligned}$$

for all $0 \le v \le w$, $w \in [c_0, \infty)$.

Proof. Follows from Lemma A.2 and Assumption 2.1. ◻

Lemma A.4. *Let*

$$R = \mathrm{e}^{Y} - 1 \quad \textit{and} \quad X = \mathrm{e}^{Z},$$

where (Y, Z) *is jointly normal distributed with mean* (μ_Y, μ_Z)*, standard deviations* σ_Y*,* σ_Z*, and linear correlation coefficient* $\rho_{(Y,Z)}$*. If* $\mathbb{E}[R] > 0$ *and* $\rho_{(Y,Z)} < 0$*, then*

$$\mathbb{E}\left[R \mid X \leq c\right] > 0 \quad \textit{for all } c \in \mathbb{R}_+.$$

Proof. The conditional distribution of Y given Z is normally distributed with mean $\mu_{Y|Z=z} = \mu_Y + \rho_{(Y,Z)} \frac{\sigma_Y}{\sigma_Z}(z - \mu_Z)$ and variance $\sigma^2_{Y|Z=z} = \sigma^2_Y(1 - \rho^2_{(Y,Z)})$, see e.g. McNeil et al. [13, p. 68]. Hence

$$\mathbb{E}[R \mid X = x] = \int_{-\infty}^{\infty} (\mathrm{e}^{y} - 1)\, \phi_{\mu_{Y|Z=\ln(x)}, \sigma^2_{Y|Z=\ln(x)}}(y) dy$$

where ϕ_{μ,σ^2} denotes the normal density function with mean μ and variance σ^2. Since $(e^{y} - 1)$ is strictly increasing in y, $\partial_x \mu_{Y|Z=\ln(x)} \leq 0$, and $\partial_x \sigma^2_{Y|Z=\ln(x)} = 0$, we infer that $\partial_x \mathbb{E}[R \mid X = x] \leq 0$. Suppose, by contradiction, there exists some $c \in \mathbb{R}_+$ with $\mathbb{E}[R \mid X \leq c] \leq 0$. Then $\partial_x \mathbb{E}[R \mid X = x] \leq 0$ implies that $\mathbb{E}[R \mid X = c] \leq 0$ and thus $\mathbb{E}[R \mid X = x] \leq 0$ for all $x \geq c$. Hence $\mathbb{E}[R] = \mathbb{E}[R \mid X \leq c]\mathbb{P}[X \leq c] + \mathbb{E}[R \mid X > c]\mathbb{P}[X > c] \leq 0$ contradicts $\mathbb{E}[R] > 0$. ◻

B Appendix: Figures and Table

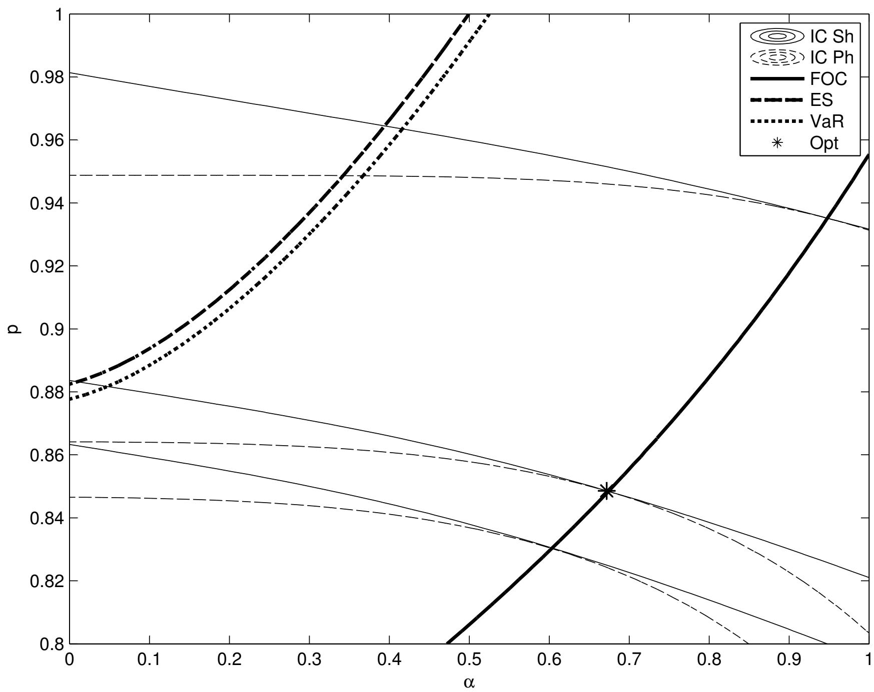

Figure 1: Numerical results for $\beta = 10$.

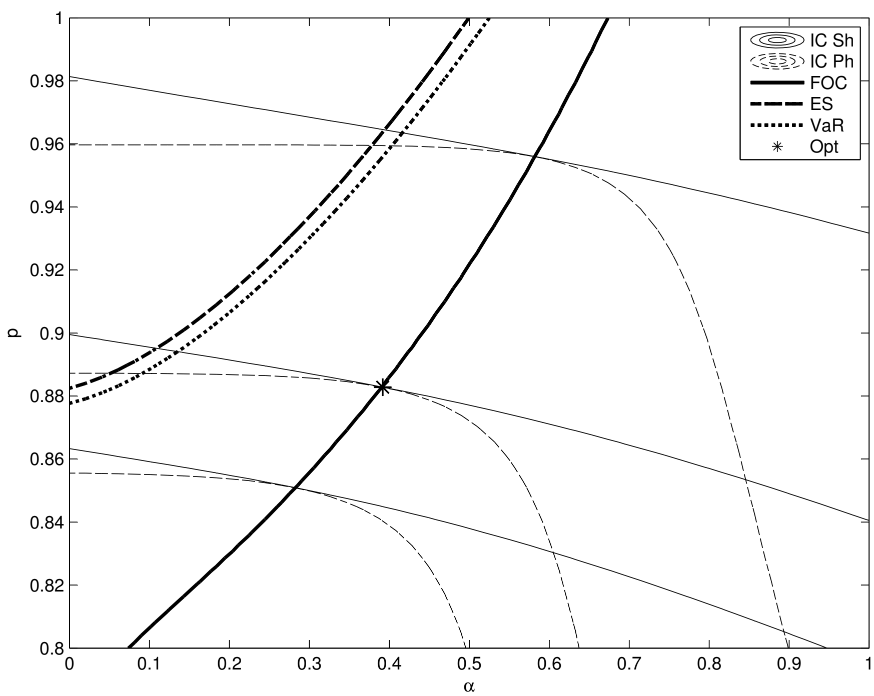

Figure 2: Numerical results for $\beta = 30$.

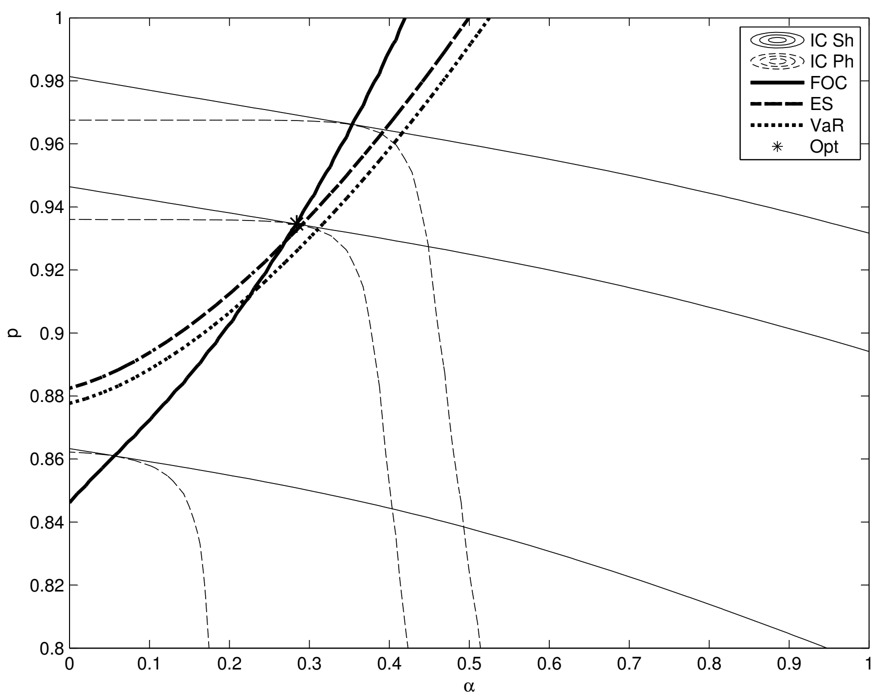

Figure 3: Numerical results for $\beta = 70$.

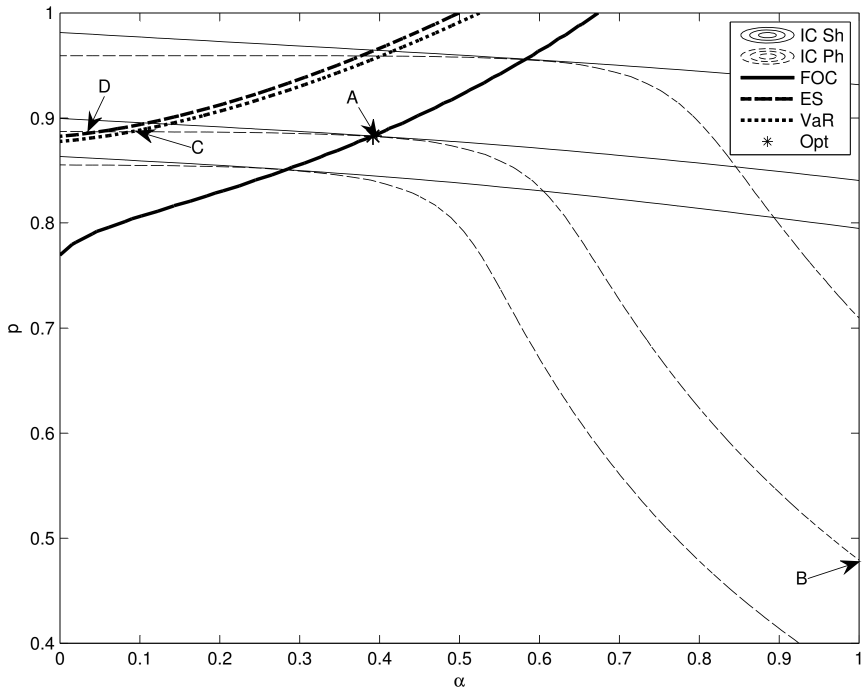

Figure 4: Numerical results for $\beta = 30$ – larger section.

	α	p	$U_{SH}(\alpha, p)$
A $= (\alpha^*, p^*)$	0.3919	0.8828	0.1465
B $= (\bar{\alpha}, \bar{p})$	1	0.4787	0.0013
C $= (\hat{\alpha}_{VaR}, \hat{p}_{VaR})$	0.0912	0.8872	0.1379
D $= (\hat{\alpha}_{ES}, \hat{p}_{ES})$	0.0516	0.8872	0.1363

Table 1: Numerical values for $\beta = 30$.

References

[1] Borch, K. (1981),“Is regulation of insurance companies necessary?,” in *Geld, Banken und Versicherungen*, ed. by H. Göpple and R. Henn., Athenäum Verlag, Königstein, 717–731

[2] Bundesamt für Privatversicherungen (2006), “Draft: Modelling of Groups and Group Effects,” URL: www.bpv.admin.ch/themen/00506/00530

[3] Committee of European Insurance and Occupational Pensions Supervisors (2007), “QIS3 Technical Specifications. PART I: INSTRUCTIONS,” URL: www.ceiops.org/content/view/118/124

[4] Committee of European Insurance and Occupational Pensions Supervisors (2007), “QIS4 Technical Specifications,” URL: http://ec.europa.eu/internal_market/insurance/docs/solvency/qis4/technical_specifications_en.pdf

[5] The Chief Risk Officer Forum (2007), “A benchmarking study of the CRO forum on the QIS III calibration,” URL: www.croforum.org/publications.ecp

[6] Doherty, N. A. (1989), “On the capital structure of insurance firms,” in *Financial models of insurance solvency*, ed. by J.D. Cummins and R.A. Derrig, Kluwer Academic Publishers, MA, USA, 267–281

[7] Finsinger, J. and Pauly, M. (1984), “Reserve levels and reserve requirements for profit-maximizing insurance firms,” in *Risk and Capital. Proceedings of the 2nd Summer Workshop on Risk and Capital*, ed. by M. Beckmann and W. Krelle, Springer-Verlag, Berlin Heidelberg, 160–180

[8] Jensen, M.J. and Meckling, W.R. (1976), “Theory of the firm: Managerial behavior, agency cost, and ownership structure,” *Journal of Financial Economics*, 3, 305–360

[9] MacMinn, R.D. and Witt, R.C. (1987), “A financial theory of the insurance firm under uncertainty and regulatory constraints,” *Geneva Papers on Risk and Insurance*, 12, 3–20

[10] Mayers, D. and Smith, C.W. (1981), “Contractual provisions, organizational structure, and conflict control in insurance markets,” *Journal of Business*, 54, 407–434

[11] Mayers, D. and Smith, C.W. (1988), “Ownership structure across lines of property casualty insurance,” *Journal of Law and Economics*, 31, 351–378

[12] McCabe, G.M. and Witt, R.C. (1980), “Insurance pricing and regulation under uncertainty: A chance-constrained approach,” *Journal of Risk and Insurance*, 47, 607–635

[13] McNeil, A., Frey, R., and Embrechts, P. (2005), *Quantitative Risk Management*, Princeton University Press

[14] Munch, P. and Smallwood, D. (1981), "Theory of solvency regulation in the property and casualty insurance industry," in *Studies in Public Regulation*, ed. by G. Fromm, MIT Press, 119–167

[15] Rees, R., Gravelle, H., and Wambach, A. (1999), "Regulation of insurance markets," *Geneva Papers on Risk and Insurance Theory*, 24, 55–68

[16] Rockafellar, R.T. (1970), *Convex Analysis*, Princeton University Press